Numerical modeling of liquefaction effects:

Development & initial applications of a sand plasticity model
Ronnie Kamai
Doctoral candidate

Katerina Ziotopoulou
Doctoral candidate
PM4-Sand:
A sand plasticity model for nonlinear seismic deformation analyses
The challenge for a constitutive model

- **Varied conditions:**
  - Loose to dense zones
  - Drained to undrained loading
  - Low to high confining stresses
  - Low to high initial static shear stress ratios

- **Common data:** $V_s$, $N_{60}$, $q_c$, gradations

![Diagram showing varied conditions with associated data points](image-url)
The challenge for a constitutive model

- Calibration to design correlations:
  - Triggering & cyclic mobility/ratcheting
  - \( G/G_{\text{max}} \) and damping
  - Strengths
  - Others depending on the structure (e.g., volumetric strains)

\[
(N_1)_{60} = 25 \\
(N_1)_{60} = 15 \\
(N_1)_{60} = 10
\]
Triggering

Equivalent clean sand corrected standard penetration, \((N_e)_{60cs}\)

- **NL**: Cases with FC \(\leq 5\%\)
- **\(\nabla\)** Cases with \(5 < FC < 15\%\)
- **\(\square\)** Cases with \(15 < FC < 35\%\)
- **\(\triangle\)** Cases with FC \(\geq 35\%\)

L: Liquefaction; NL: No Liquefaction

Curve recommended by Idriss & Boulanger (2004) for clean sands

Number of cycles to cause initial liquefaction

Cyclic triaxial tests with \(D_r = 70\%\):
- \(\alpha_{\text{cyc}}/\sigma' = 2 \& 16\) (Vaid & Chem 1985)

Simple shear tests with \(\alpha_{\text{cyc}}/\sigma' = 2\):
- \(D_r=35 \& 55\%\) (Boulanger et al. 1991)
- \(D_r=50 \& 68\%\) (Vaid & Finn 1971)

Vertical effective stress, \(\sigma_{vc}/\sigma_a\)

Shaking table tests conducted by De Alba et al (1976):
- \(\Delta\): Relative Density = 90%
- \(\bigcirc\): Relative Density = 82%
- \(\bigtriangledown\): Relative Density = 68%
- \(\bigtriangleup\): Relative Density = 54%

Initial confining pressure = 8 psi (55 kPa)
Plasticity model for sand – Starting point

  - Critical state, stress-ratio based
  - Bounding and dilation surfaces rotate with changes in state
  - Fabric tensor used to enhance contraction rates
Modified & calibrated at equation level to approximate design correlations for practice

- Modified fabric tensor to depend on plastic shear strains
- Added fabric history, including cumulative fabric term
- Plastic modulus ($K_p$), elastic moduli (G), and dilatancy (D) depend on fabric and fabric history
- $D$ constrained by Bolton's (1986) dilatancy relationship
- Recast in terms of relative state parameter index ($\xi_R$)
- Inclusion of sedimentation effects
- Modified logic for updating initial back-stress ratio
- Neglects Lode Angle dependence

Implemented as a user-defined material model in FLAC (Itasca 2010)
Practical means for including critical state framework

Relative density, $D_R$

Critical state line from $I_{RD}$ relation (Bolton 1986) with $Q=10$ & $R=1.5$

$$\xi_R = D_{R,CS} - D_R$$

$$D_{R,CS} = \frac{R}{Q - \ln \left( \frac{100(1 + 2K_0)\sigma'_{vc}}{3p_A} \right)}$$

Mean principal effective stress, $p/p_A$
Stress ratio based

- Dilatancy & bounding surfaces collapse to $M$ at critical state ($\xi_R = 0$)

$$M^b = M \cdot \exp\left(-n^b\xi_R\right) \quad M^d = M \cdot \exp\left(n^d\xi_R\right)$$
Fabric effects

- **Fabric**
  
  \[ dz = -\frac{c_z}{1 + \left(\frac{z_{\text{cum}}}{2z_{\text{max}}}ight)} \frac{\langle -d\varepsilon^p_v \rangle}{D} (z_{\text{max}}n + z) \]

- **Elastic modulus**
  
  \[ G = G_o p_A \left(\frac{p}{p_A}\right)^{1/2} C_{SR} \left(1 + \frac{z_{\text{cum}}}{4z_{\text{max}}} \right)^{0.5} \]

- **Plastic modulus**
  
  \[ K_p = G \cdot h_o \cdot \frac{\left([\alpha^b - \alpha]:n\right)^{0.5}}{\exp\left([\alpha - \alpha_{in}]:n - 1\right) + C_{y1}} \cdot \frac{C_{K\alpha}}{1 + C_{Kp} \frac{Z_{\text{peak}}}{Z_{\text{max}}}} \langle [\alpha^b - \alpha]:n \rangle \]

- **Dilatancy**
  
  \[ D = A_d \cdot \left([\alpha^d - \alpha]:n\right) \]
  
  \[ A_d = \frac{A_{do} (C_{z\text{in}2})}{\left(\frac{Z_{\text{cum}}}{Z_{\text{max}}}\right)^3 \left(1 - \left\langle -z:n\right\rangle / \sqrt{2 \cdot Z_{\text{peak}}}\right) (C_{e}) (C_{pzp}) (C_{p_{min}}) (C_{z\text{in}1}) + 1} \]

  \[ D = A_{dc} \cdot \left([\alpha - \alpha_{in}]:n + C_{in}\right)^2 \frac{\left(\alpha^d - \alpha\right):n}{\left(\alpha^d - \alpha\right):n + C_D} \]
  
  \[ A_{dc} = \frac{A_{do}}{h_p} \]
  
  \[ A_{dc} = f\left(Z, \varepsilon_R, \ldots\right) \]
Functionality versus simplicity

- Simple parts, easy to understand
- Rides well
Parameters

- **Relative density** ($D_R$)
  - Estimate from SPT or CPT; adjusts stress-strain responses

- **Shear modulus coefficient** ($G_0$)
  - Calibrate to in-situ $V_s$ data or correlations

- **Contraction rate parameter** ($h_{po}$)
  - Calibrate to CRR estimated from SPT- or CPT-based liquefaction correlations

- **Secondary parameters**
  - 18 secondary parameters with default values chosen to approximate design correlations
Example responses

Shear stress ratio, \( r/\sigma'_{vc} \)

\( D_R = 35\% \)
\( \sigma'_{vo} = 100 \text{ kPa} \)

\( D_R = 55\% \)
\( \sigma'_{vo} = 100 \text{ kPa} \)
Example responses

![Graph 1](image1.png)

- **$\sigma'_{\text{vo}} = 100$ kPa, $r_u > 98\%$**
- **$D_R = 75\%$**
- **$b = 0.30$**
  - 55%
  - 35%

![Graph 2](image2.png)

- **$\sigma'_{\text{vo}} = 100$ kPa, 3% shear strain**
- **$D_R = 75\%$**
- **$b = 0.36$**
  - 55%
  - 35%
Example responses

\[ D_R = 55\% \]
\[ \sigma'_v = 100 \text{ kPa} \]

\[ D_R = 55\% \]
\[ \sigma'_v = 400 \text{ kPa} \]
Site response of Port Island and Wildlife Liquefaction Arrays
Wildlife liquefaction array

Data from Bennett et al. 1984, Holzer & Bennett 2010 personal comm.
WLA response in 1987 Superstition Hills Eq.

- **Surface motion**

![Graphs showing acceleration and SA over time and period](image-url)
Centrifuge test with lateral spreading
Centrifuge model SSK01

[NEES test by Kamai, Kano, Conlee, Marinucci, Boulanger, Rathje, Rix, and Howell 2008]
Calibration

- $V_s$ measured in the model
- CRR from lab tests
Input motion: Sequence of progressively stronger shaking events, each being 20 cycles at 2Hz
Excess pore pressures

**Shake 09**

- FLAC Case B
- FLAC Case A
- SSK01

**Shake 10**

$\sigma'_0 \approx 15 \text{ kPa}$

**Shake 11**

$\sigma'_0 \approx 25 \text{ kPa}$

$\sigma'_0 \approx 35 \text{ kPa}$

Input acc (g)

Time (sec)
Strain concentration beneath clay crust
Strain concentration beneath clay crust

Shake 10, Case B

X displacement

Shake 10, Case B

Relative density
Centrifuge test of slope with silt interlayers
Centrifuge test of slope with silt interlayers

- Nevada sand, $D_R \approx 35\%$

(Malvick, Kutter, & Boulanger 2008)
Initial stresses
Accelerations

PM4Sand, $D_R=35\%$

pt. C

pt. B

pt. A

EJM02_e2  Case A  Case B
Excess pore pressures

PM4Sand, $D_R=35\%$

$\Delta U$ (kPa)

$\Delta U$ (kPa)

Input acc (g)

$\Delta U$ (kPa)

Time (sec)

Case A

Case B

EJM02_e2
Strains & displacements

Shear strain $\gamma$ (%)

Volumetric strain $\varepsilon_v$ (%)

Lateral displacement (m)

Baseline, PM4Sand, 80sec
Strain concentration at silt seam

- Influence of localization scale, permeabilities, re-sedimentation strains and other factors.
Strain concentration at silt seam

Influence of localization scale, permeabilities, re-sedimentation strains and other factors.
**Concluding remarks**

- **PM4-Sand** is a stress-ratio controlled, critical state compatible, bounding surface plasticity model with fabric which was developed and calibrated to approximate trends in design correlations commonly used in the USA.

- Initial applications of PM4-Sand have been promising, suggesting that it reasonably approximates the principle behaviors of liquefying sands.

- **Numerical analyses of liquefaction effects**
  - can provide valuable insights regarding complex mechanisms of behavior, but
  - can have significant bias and dispersion in computed responses depending on the specific problem (and on the numerical procedures & calibration protocols).

- Dynamic centrifuge model studies provide a valuable basis for systematically evaluating numerical analysis methods.
Support

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Thank you.
Strains & displacements

Case B, $k_{base} \times 10$
PM4Sand, 55sec

Shear strain $\gamma$ (%)

Volumetric strain $\varepsilon_v$ (%)

Lateral displacement (m)

PM4Sand, 55sec
Localization scales in the field?
Localization scales in the field?

(modified after Naesgaard et al. 2006)

a) Continuous water film
b) Venting + collapse of water film
c) Undulating surface
d) Spatial discontinuity of barriers
Example responses

Plane strain compression:
\[ \phi_{cv} = 33^\circ, K_0 = 1.0 \]

\[ \phi' = 2 \tan^{-1} \left( \frac{\sigma'_3}{\sigma'_1} \right) - 45^\circ \]

\[ D_R = 75\% \]
\[ D_R = 55\% \]
\[ D_R = 35\% \]
Example responses

Undrained DSS, \( D_R = 35\% \)

\( \sigma'_v = 0.25, 1, 4, \) & 16 atm

Shear stress (kPa)

Shear strain (%)

Vertical effective stress (kPa)

Critical state
Port Island Array, Kobe

[Data from]
PIA response in 1995 Kobe Earthquake

- Surface motion