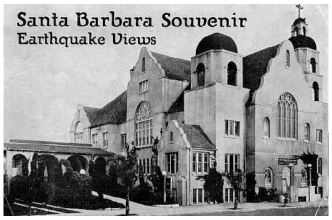

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APPLICATION OF MARKOV CHAIN MONTE CARLO METHOD IN INVERSION OF SURFACE-WAVE PHASE VELOCITY

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ABSTRACT

Heuristic algorithms, such as GA and SA have been frequently used in the inversion of surface-wave phase velocity in microtremor explorations. One of the advantages of the heuristic inversion methods is the no requirement of calculation of derivatives of an objective functions and matrix inversions. Therefore the inversions based on the algorithms are so robust. Although the heuristic methods can globally search model parameters in parameter spaces, it is sometimes difficult to estimate model sensitivity in these approaches. In this study Markov Chain Monte Carlo method is applied in surface wave phase velocity inversion for microtremor exploration. The algorithm is one of numerical statistical approaches for calculating a probability density function of inversion parameters based on likelihood function from observed data. I first explained the computational procedure of the method. Then, the applicability of the method is examined using synthetic phase velocity data of Rayleigh wave. Resolutions for model parameters were estimated using sampled models from the method. I furthermore estimated variations of predominant periods and amplification factors for the sampled models. It is concluded that this method can be used for the phase velocity inversion similar to the other heuristic methods, but can provide resolutions of parameters of inverted models. I finally applied the method to actual Rayleigh wave phase velocity data from microtremor explorations in the Tokyo basin.

INTRODUCTION

It is known that S-wave velocity distribution in depth is one of the crucial factors to estimate the effects of surface geology on seismic motion during an earthquake. Various kinds of geophysical exploration techniques have been developed to know S-wave velocity profile in shallow and deep soils. Although an exploration using a bore hole can provide the most reliable data on an S-wave velocity profile, it requires a lot of expenditure in field operation. Furthermore it is sometimes difficult to conduct borehole loggings in an urban area. One of the most inexpensive explorations is the technique using microtremors. In particular microtremor array exploration is capable to provide a 1D S-wave velocity profile from records of microtremors in an array deployed on the surface. Because of the easy field operations, this technique is widely used in geophysical explorations for evaluation of the local site effects in various kinds of soil conditions (e.g., Okada, 2003). It is also noted that the microtremor array exploration is the only exploration technique in which frequency range of wave motion used is similar to those of engineering interest.

The microtremor array technique is principally based on estimation of phase velocity of surface wave (mainly Rayleigh waves) and its inversion to a 1D S-wave velocity profile. Frequency–wave number spectral analysis and spatial autocorrelation method have been used in the retrieving Rayleigh-wave phase velocity from array records of vertical microtremors (Okada, 2003). Least square approaches are traditionally used in phase velocity inversion in microtremor array exploration. It is well known that the least square methods have some difficulties in practical applications, such as numerical instability and trapping at local minimum models. In particular, the existence of local minimum solution leads to the dependence of final solution to initial model assumed in advance. This practical difficulty in the phase velocity inversions can be solved with an application of heuristic approaches. The heuristic approaches in the inversions are one of Monte Carlo methods, but models with small misfit are effectively searched using various kinds of the algorithms. One of the typical approaches is genetic algorithms and simulated annealing (e.g., Yamanaka, 2005). Since these methods do not use the derivatives of misfit function to be minimized, a chance for trapping of solution at local minimum models is smaller than those in the conventional least square methods. Therefore these methods have been widely applied in actual inversions of surface-wave phase velocity in actual microtremor exploration work (e.g., Yamanaka et al., 2000). One of the disadvantages of these heuristic inversions is a large computational time required. Furthermore, an understanding a model resolution which is usually obtained in the conventional least square methods is not easily obtained in the heuristic methods.

In this study Markov Chain Monte Carlo method is applied in surface wave phase velocity inversion for the microtremor exploration. I first explained the computational procedure of the method. Then, the applicability of the method is examined using synthetic phase velocity data of Rayleigh wave. I finally applied the method to actual Rayleigh wave phase velocity data from the microtremor explorations in the Tokyo basin.

MCMC METHOD

The Markov Chain Monte Carlo method is one of numerical statistic approaches where the probability density function for the target distribution is numerically estimated using random sampling of parameters. It is significantly difficult to sample randomly wide parameter spaces with conventional Monte Carlo methods. The MCMC method is however capable to sample a complex function having many unknown parameters with reasonable computational cost with am assumption of stationary Markov chain condition. The method is originally used for numerical estimation of probability density functions of arbitrary functions, and applied in various kinds of science and technologies (e.g., Gilk et al., 1996).

The MCMC methods reply on the Bayes theorem. According to the theorem, the probability density function $p(\mathbf{m}/\mathbf{d})$ of model parameters, $\mathbf{m}$, with given data, $\mathbf{d}$, is derived from

$$p(\mathbf{m} \mid \mathbf{d}) = p(\mathbf{d} \mid \mathbf{m}) p(\mathbf{m}) / p(\mathbf{d}). \tag{1}$$

Here $p(\mathbf{m})$ is a prior distribution of the model parameters, and $p(\mathbf{d})$ is the distribution of the data. p(d/m) is a conditional probability density function of $\mathbf{d}$ for given $\mathbf{m}$, and corresponds to a likelihood function. $p(\mathbf{m}|\mathbf{d})$ is called posterior distribution of the model parameters. Because we usually have the data before the inversions, the probability distribution of data $p(\mathbf{d})$ is constant. Then equation (1) is written in the form,

$$p(\mathbf{m} \mid \mathbf{d}) \propto p(\mathbf{d} \mid \mathbf{m}) p(\mathbf{m}). \tag{2}$$

This equation shows how the distribution of the model parameters is modified after having the data. The likelihood function in our inversion of the surface-wave phase velocity can be expressed by

$$p(\mathbf{d} \mid \mathbf{m}) = L(\mathbf{m}) = \exp[-E(\mathbf{m})], \tag{3}$$

where E(m) is a misfit function defined using the least-squared differences between the observed phase velocity, $C^o(T_i)$, and calculated phase velocities, $C^c(T_i)$, as

$$E(\mathbf{m}) = \sum_i \left[\frac{C^o(T_i) - C^c(T_i)}{\sigma(T_i)} \right]^2 . \tag{4}$$

Here $\sigma(T_i)$ is a standard deviation of the observed phase velocity at a period of Ti. When we do not have a prior information on model parameters, $p(\mathbf{m})$ is a uniform distribution. The equation (3) can be simplified as

$$p(\mathbf{m} \mid \mathbf{d}) \propto p(\mathbf{d} \mid \mathbf{m}) = \exp[-E(\mathbf{m})] \tag{5}$$

The distribution of the model parameters can be estimated from this equation using the misfit function. When we have a prior knowledge on subsurface structure from previous explorations, we can use the equation (2). It is noted that the inversion based on this approach does not provide the best model that fits the observed phase velocity with the minimum errors, but a distribution of model the parameters. Therefore we can estimate an accuracy of the model parameters together with the model parameters which can be the average of the models of the distribution. It is not usually an easy task to evaluate the distribution by drawing samples of model parameters. One of the ways for this evaluation is through a Markov chain as its stationary distribution. Markov chain is known as sampling state where the sampling result only depends on the current model, and does not depend on history of the sampling. Assuming the Markov chain, the sampling results converge to the distribution of the parameters. We apply the numerical procedure based on Metropolis-Hasting algorithm (Hastings, 1970) to construct a Markov chain for estimation of the stationary distribution of the equation (5).

Suppose that $\mathbf{m}_i$ is the current model at i-th iteration of the chain. A model, **m'**, is generated using a proposal distribution q(.|$\mathbf{m}_i$). We used the normal distribution with an average of $\mathbf{m}_i$ and standard deviation of $\boldsymbol{\sigma}_i$ as the proposal distribution. The standard deviation can control the generation of the new model. The acceptance of the generated model is determined with the following acceptance criteria. The acceptance probability, *r*, is defined as

$$r = \min\left(\frac{L(\mathbf{m}')q(\mathbf{m}_i, \mathbf{m}')}{L(\mathbf{m}_i)q(\mathbf{m}', \mathbf{m}_i)}, 1 \right). \tag{6}$$

If the candidate model, **m'**, is accepted, the next model in the chain is $\mathbf{m}_{i+1} = \mathbf{m}'$. If the candidate model is rejected, the current model is kept; namely $\mathbf{m}_{i+1} = \mathbf{m}_i$. The numerical operation in this acceptance is similar to that in the Metropolis simulated annealing (e.g., Yamanaka 2005).

The sampling process with the MCMC method in a finite length of iterations more or less depends on initial model assumed. It is known that the MCMC algorithms can generate a stationary sampling after long-term iterations. Therefore some of the sampled data must be rejected before it reaches a stationary state of a Markov chain. The term for the rejection of the data sampled before the stationary sate is called burn-in period. In order to determine the burn-in period for extracting the stationary part of the sampled data, I used a convergence criteria by Geweke (1992). From the sampled data with enough many iterations, the average (g_1) and standard deviation (σ_1) for the first *n* data are calculated. I also calculated those (g_2 and σ_2) for the *m* data from the end of the iteration. Then Z-value is calculated as

$$Z = (g_1 - g_2) / \sqrt{\sigma_1 - \sigma_2}\ . \tag{7}$$

It is indicated that the sampling is convergent with a significant level of 5%, if the Z-value is less than 1.96 (Geweke, 1992). In this study, *n* and *m* are set to be 10% and 50 % of the number of the sampled data after deleting the data in the burn-in period.

NUMERICAL TESTS

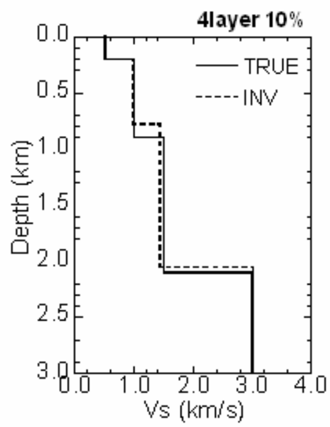
The inversion based on the MCMC method is examined using synthetic phase velocity for Rayleigh wave. In particular we focus on the phase velocity inversion for profiling an S-wave velocity profile of deep sedimentary layers of a large basin, such as Kanto basin, Japan, LA basin in California, and so on.

S-wave Velocity Model and Synthetic Data

The S-wave velocity model shown in Figure 1 and Table 1 are used to generate synthetic phase velocity data for fundamental Rayleigh waves in the numerical test. This is one of typical S-wave velocity models of the deep sedimentary basin, such as Kanto basin in Japan (e.g., Yamanaka and Yamada, 2006). I calculated fundamental Rayleigh-wave phase velocity for the model at periods from 0.1 to 2Hz. Then the phase velocity is contaminated with random noises whose are generated from a uniform distribution having amplitude of 5% of the phase velocity. The standard deviation for the synthetic phase velocity is also assumed to be 10 % of the phase velocity. The resultant phase velocity is displayed in Figure 2. The S-wave velocity and the thickness for each layer in the 4-layer model are optimized in the inversion of the phase velocity. P-wave velocity is given from S-wave velocity using an empirical relationship by Kitsunezaki et al (1996). The density for each layer is given in advance.

Table 1 Model parameters and search limits for numerical test.

Vs (km/s)		Thickness (km)		Density (g/cm^3)
true	Search limits	true	Search limits	
0.5	0.2-1.6	0.2	0.02-1.4	1.8
1.0	0.4-1.8	0.7	0.05-1.6	1.9
1.5	0.6-2.5	1.2	0.1-2.	2.0
3.0	2.2-3.5	-	-	2.3



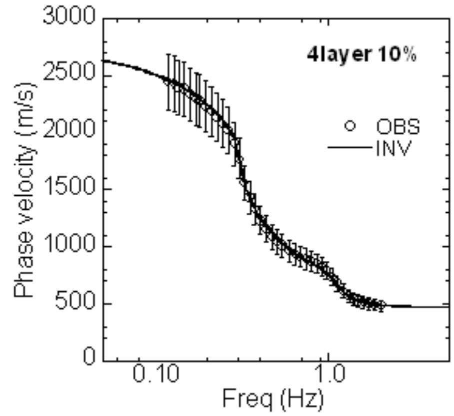


Fig. 1. True and inverted S-wave velocity models.

Fig. 2. Synthetic Rayleigh wave phase velocity for the true model in Fig.1.

Estimation of Burn-in Period

An initial model is randomly generated within the search limits in Table 1. Then, the initial model is modified through the MCMC sampling. Figure 3 shows the variations of the S-wave velocity and thickness of each layer. The parameters for the initial 5000 models are different from the true values showing a dependency on the initial model assumed. Then, the parameters for the first and second layers seem to converge around the true values showing a stationary state of the Markov chain. The other parameters vary gradually to the true values, and converge after 70000 iterations.

The Z-values are calculated for different burn-in periods as can seen in Figure 4. The Z-values in the figure indicates the maximum value among the Z-values for all of the parameters. The Z-value rapidly decreases beyond the burn-in period of 70000 models, and it becomes less than 2.0 after 80000 models were removed as the burn-in period. Therefore, we can use the models beyond this burn-in period to evaluate the distribution of the model parameters.

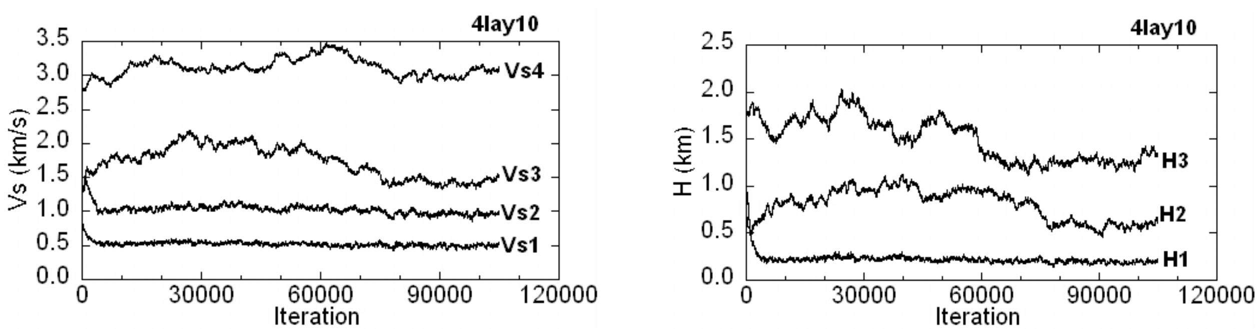


Fig. 3. Variation of S-wave velocity and thickness from MCMC sampling for synthetic phase velocity in Fig.2.

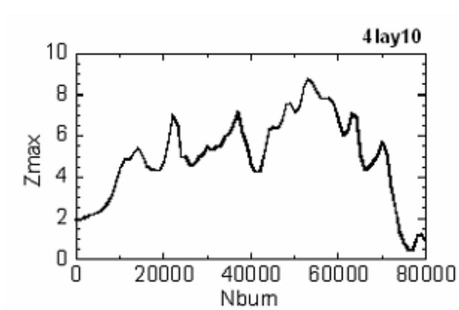


Fig. 4. Relationship between Z-values and burn-in period.

Results of Inversion

The distributions of the S-wave velocities and the thicknesses are evaluated for the sampled models after the burn-in processing as shown in Figure 5. The average values and the standard deviations for the parameters are also shown in the figure. The distribution of the S-wave velocity and the thickness for the top layer is very narrow and sharp indicating a high resolution, while those for the deep part are wider distribution than those for the top layer. The relatively low resolution is due to the lack of the phase velocity at periods lower than 0.1 Hz. However, the distributions of all the parameters are enough to determine the final inverted model. The inverted model shown by a broken line in Figure 1 was derived from averaging the parameters of all the models sampled with the MCMC method. The two models are quite similar, though a slight differences of the depth to the interface at a depth of about 1km. Since we assumed the small standard deviation for the observed data, the standard deviation of the estimated parameters is also very small. The phase velocities for the inverted model are also compared with the observed data in Figure 2. The observed phase velocities at all the periods are well explained by the theoretical phase velocity.

It is well known in the least-squares inversion theory that the errors in the observation determine the accuracy of the parameters. Here we investigate the effects of the observational errors assumed on the estimated errors of the parameters. First we generated the synthetic Rayleigh waves with the different standard deviations from 5 to 40 % in the similar manner explained above. Then the similar samplings were conducted for each synthetic data. Figure 6 display the relationship between the standard deviations for the observed phase velocity and parameters for the sampled models. It is noted that each value is normalized with its average to obtain variation coefficients. The coefficients for the parameters in the shallow part are more affected by the observational errors than the deep part. This figure also indicates that the observed phase velocity must be obtained with an accuracy of 30 % for an estimation of S-wave velocity in the top layers with an accuracy of 10 %. Similarly we can see that high accuracy phase velocity data are needed for estimating the thickness with the same accuracy. This numerical test clearly indicates the advantage of the inversion based on the MCMC method for estimation of not only an inverted profile but also its accuracy from a probability distribution of the model parameters.

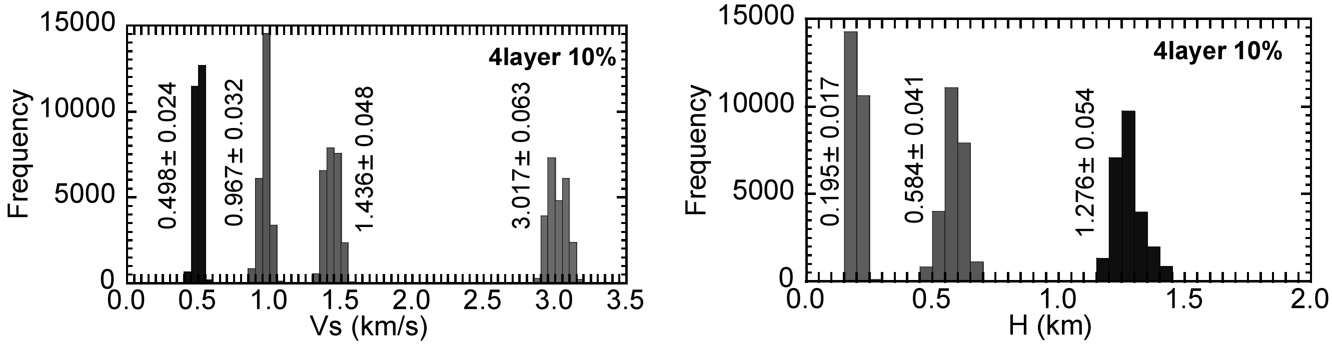


Fig. 5. Distributions of S-wave velocity (left) and thickness (right) from sampled models.

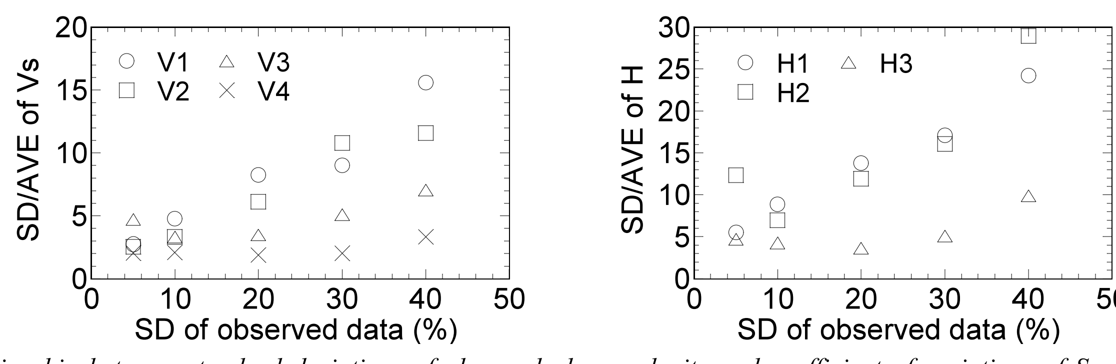


Fig. 6. Relationship between standard deviations of observed phase velocity and coefficient of variations of S-wave velocity and thickness of the sampled data.

ESTIMATION OF VARIATIONS OF AMPLIFICATIONS

Once we estimated the variation of the parameters for the inverted S-wave profile, the variation of the amplification factors can be also obtained numerically. Here I calculated 1D amplification factors for the vertically propagating S-waves in the 1D models sampled in the MCMC method. Figure 7 shows the distributions of the maximum amplification and predominant peak periods of the amplification factors for the inversion of the phase velocity with the standard deviation of 10 %. The distribution of the maximum amplitudes and peak periods are very narrow and its standard deviation is just 2% of the averaged values that is the same as the true values. A slightly broad trail part in the distribution of the amplification amplitude in the high value side is probably due to the low accuracy of the deep part of the model. Similar calculations of the amplifications were conducted using the sampled models for the phase velocities with the different observational errors. The relationship between the standard deviation of the observed phase velocities and the coefficient of variations for the predominant peak period and the maximum amplitudes are shown in Fig.8. It can be seen that the peak period and the maximum amplitudes are insensitive to the observation errors. Since the maximum amplitude of the amplification factors is mainly controlled by the impedance contrast between the top layer and the basement, the low values of the variation coefficients are due to the high accuracy of S-wave velocity in the shallow part of the models.

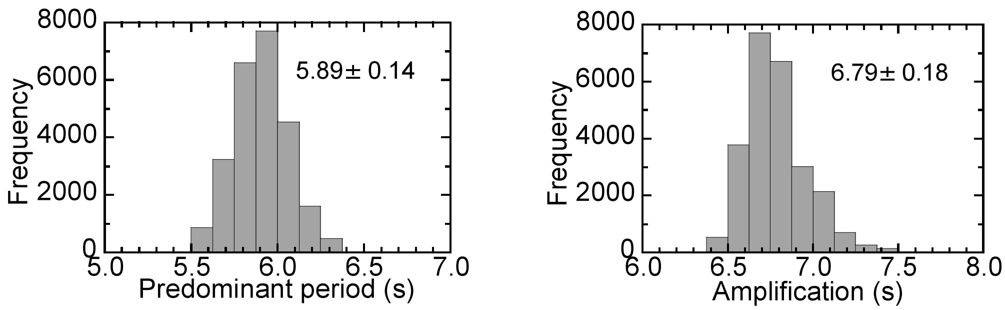


Fig. 7. Distributions of predominant period and maximum amplitudes of 1D amplification factors for sampled models.

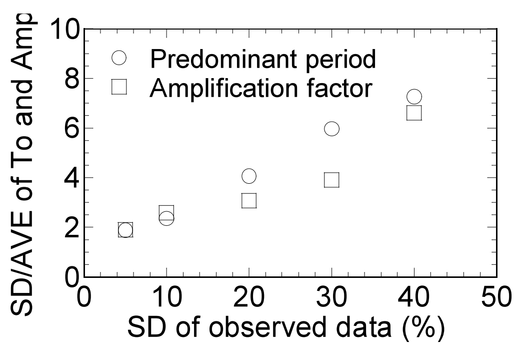


Fig.8. Relationship between standard deviations of observed phase velocity and coefficient of variations of predominant period and maximum amplitude of 1D amplification factors for the sampled data.

APPLICATION TO ACTUAL PHASE VELOCITY

I apply the MCMC inversion method to Rayleigh wave phase velocity from actual microtremor array exploration in the Kanto basin. Here, the phase velocity at ASO in Yamanaka et al. (2000) is used as shown in Figure 9. They inverted the observed phase velocity with a genetic algorithm to derive a 1D S-wave velocity profile that is also shown in Figure 9. The MCMC inversion was conducted assuming a 4-layer model. The inverted S-wave velocity profile is derived from averaging the parameters for all the sampled models after the burn-in operation. The inverted profile is almost the same as that from the genetic inversion. The comparison between the observed and the theoretical phase velocities can be seen in Figure 9. The distributions of the S-wave velocities and the thicknesses for the sampled models are shown in Figure 10. The parameters for the top layer are well resolved with the narrow distributions. The S-wave velocity for the second layer is also inverted with a high accuracy, while the distribution of its thickness is broad showing a low resolution. Furthermore the parameters for the third layer and basements distribute in wide ranges. The lack of the phase velocity data at period more than 4 seconds is the main reasons for the low resolutions of the parameters of the third layer.

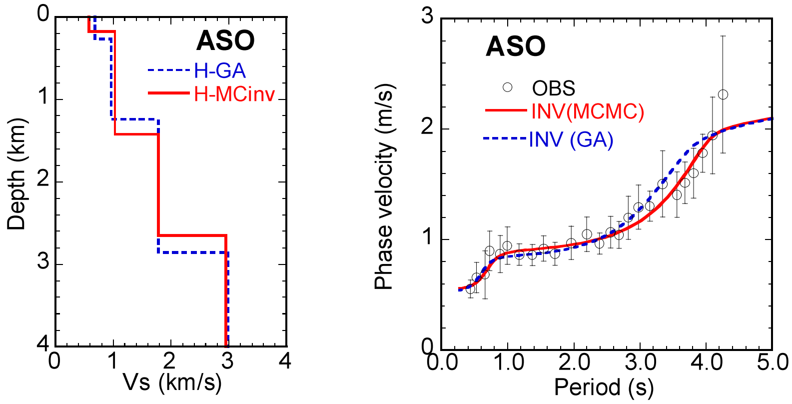


Fig. 9.Comparison of inverted results from previous study and this study. Left is S-wave velocity profiles and right is phase velocities for the inverted models with observed ones.

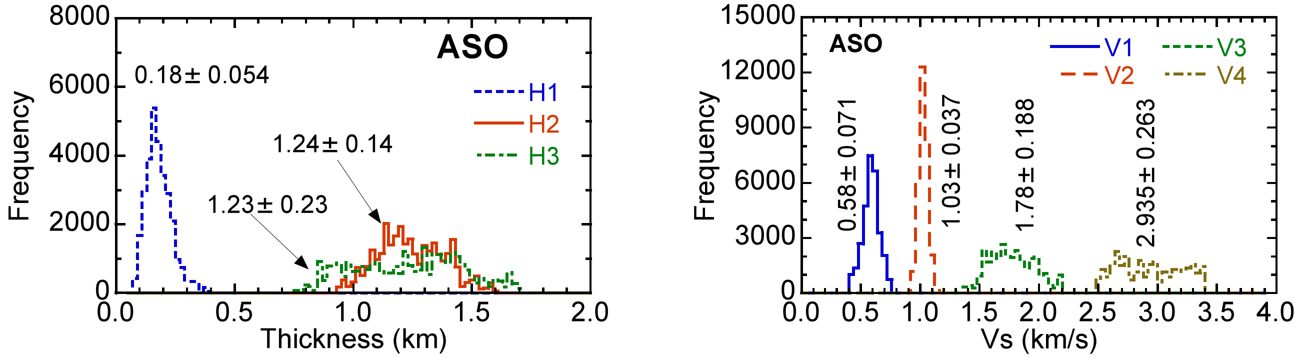


Fig.10. Distributions of S-wave velocity (left) and thickness (right) from sampled models by MCMC inversion of the observed phase velocity at ASO in Figure 9.

CONCLUSIONS

I applied the Markov Chain Monte Carlo method to surface wave phase velocity inversion. The numerical test of the method was conducted using the synthetic Rayleigh wave phase velocity data. The method can evaluate not only an inverted S-wave velocity profile, but also probabilistic distributions of the model parameters from which we can understand model resolutions. This can be addressed as one of the powerful advantages for the method. Since the MCMC inversion method requires only the calculation of the misfit function just like genetic algorithms and simulated annealing, this method is also very robust during computations. I also apply this method to the observed phase velocity data from actual microtremor explorations in the Kanto basin. The results of the inversion clearly indicated a high resolution of the S-wave velocity and the thickness for the shallow part and a low resolution for the deep part, because of the absence of phase velocity at long-periods.

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